PILOT AUCTION FACILITY FOR METHANE AND CLIMATE CHANGE MITIGATION: AUCTION DESIGN

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POWER AUCTIONS

Contents

Terminology

Term	Description
Activity rule	The rule that limits what bids a bidder can make in subsequent rounds of a multiple round auction based on the bidder's bids in earlier rounds. The activity rule is intended to avoid bid sniping. A bidder with a large demand late in the auction must express a large demand in the earlier rounds when the strike price is higher.
Aggregator	A firm/bidder that wants to purchase put options in order to fund a large number of methane abatement projects which it is not implementing itself.
Bid sniping	The tendency to wait until the last possible opportunity to place a serious bid as in an eBay auction. Auctions often have activity rules in place to prevent bid sniping.
Cap	A competition constraint rule that would prohibit a single bidder from winning more than some percentage (e.g. 25%) of the available supply or budget.
Clearing price[1]	The price at which the demand for a put option is no longer above its supply, and thus is the price when the put option "clears." This clearing price may be more than the winner's bid price.
Clock auction	A multiple round auction in which in each round the auctioneer announces prices and the bidders respond with demands at the specified prices. Prices then decrease on products with excess demand and the process repeats.
Clock price	A price for a put option in a round of a clock auction. Also called the round price.
Closing rule	The rule that describes the conditions for ending the auction.
Collusion	Two or more bidders working together to manipulate the auction outcome.
Competition constraint	A rule designed to achieve social goals such as encouraging competition in a given area. Competition constraints may be implemented in a number of ways, such as setting caps.
Dynamic auction	Any auction format that involves multiple opportunities to bid and where some information about the bidding is revealed to the bidders during the course of the auction.
Information	The policy that determines the information that is revealed to bidders during

[1] Price either refers to the put option's premium (in the case of a forward auction) or the strike price (in the case of the reverse auction). In this glossary, price refers to the strike price but these concepts apply similarly in a forward auction. See companion paper on relevant auction theory for a glossary where price refers to the premium.

policy	the course of a dynamic auction. The information revealed might include bid-specific information such as the price of the bid and the identity of the bidder, or aggregate information such as the total number of bids made on a certain product (demand for that product).
Intra-round bids	Bids submitted at prices between clock prices, identifying at exactly what price a bidder wants to change his/her demand. These bids help smooth out the aggregate demand curve to avoid undersell.
Price-point	The percentage of the price decrement (between the clock price of the previous round and the clock price of this round) that the price of an intra-round bid corresponds to.
Price discovery	A feature of dynamic auctions in which information about bidder demands is reported to bidders, giving bidders the opportunity to adjust subsequent bids based on the information.
Price decrement	In a reverse clock auction, this is the amount by which the price decreases between rounds.
Pricing rule	The rule that determines the price paid by the bidder for each put option that it has won.
Proxy bid	A mechanism by which a bidder may submit a bid ahead of time before the auction reaches a given price. The proxy bid is automatically entered into the system when certain conditions are met.
Reserve price	The maximum strike price for which PAF is willing to sell a put option (given a fixed premium).
Sealed-bid auction	An auction in which bidders submit bids without receiving any information relating to the bids placed by other bidders.
Set-aside	A competition constraint rule that sets aside or “carves out” specific items for bidders meeting certain criteria. A set-aside is sometimes used for new entrants in a market where new entry is desirable to increase competition. Alternatively, a set-aside can be used for small firms or for projects in specific countries or sectors.
Supply	The number of put options being sold by the Pilot Auction Facility as a function of the price.
Winner Determination	The process of deciding who to award what at the end of the auction.
Undersell	The event in which one or more put options remain unsold when the auction concludes.

1. Overview

The Pilot Auction Facility for Methane and Climate Change Mitigation (PAF) is developing plans for incentivizing the reduction of methane emissions in developing countries by auctioning put options to project implementers or third parties. Such a contract gives the holder the right, but not the obligation, to sell a ton of CO_2e in achieved methane emission reductions at a given "strike" price.

This document outlines our recommendations for the design and implementation of these auctions. This includes a detailed description of the auction format, how the auction should be run, what bidder support and training should be offered before and during the auction, and what information should be released after the auction. The companion document *Pilot Auction Facility for Methane and Climate Change Mitigation: Relevant Auction Theory* describes different auction formats and the advantages of the clock format that we are recommending for the PAF.

This document also includes details adapting the procedure for either a forward or reverse format. In a forward auction, the strike price (the price the PAF pays when the put option matures) is fixed and bidders bid up the premium (the price bidders pay for the put option). In a reverse auction, the premium is fixed and bidders bid down the strike price.

2. Recommendation

2.1. Auction Format

We recommend using an (ascending or descending) clock format to sell these put options. The many benefits of using this approach are outlined in the companion document on relevant auction theory. To briefly summarize, the clock auction:

- Enables price discovery. Bidders can incorporate information from others bids into their own bidding strategy, and the final price incorporates all this information.
- Simple and transparent for bidders. Bidders are only asked for a small amount of information in each round, so bidding in each round is not overly complex. Also, the process that sets the final price is transparent and easy to understand. These make participation in the auction easy for bidders.
- Accommodates a variety of auction designs. A clock auction easily accommodates both the forward and reverse designs. Moreover, the clock auction format can be used to sell multiple types of put options in a single auction, should the PAF decide to do so.
- Encourages truthful bidding. Bidders do not have strong incentives to strategically manipulate their bids. This makes bidding easier for bidders and makes it more likely that the bids and the final price reflect true market conditions.

2.2. Number of Auctions

We recommend that the PAF uses a relatively small number of auctions ("auction rounds"). The supply in each auction should be sufficiently large that a bidder could win all the options he needs in one auction and would perceive a reasonable probability of winning. A reasonable minimum budget per auction seems to be about $25 million. Since participation in an auction is costly, a bidder may be less likely to participate if he believes that he will need to participate in multiple auctions in order to win the number of put options he wants. For a total budget of $100 million, we therefore recommend conducting 2 to 4 auctions.

2.3. Reverse or Forward Auction

We recommend a reverse auction, where the premium price is fixed and bidders bid on the strike price, for the first auction. Bidders are likely to be more comfortable with this format in this context and with a sufficiently low premium this format could lead to greater participation than a forward auction. Since one of the purposes for the PAF is learning, we suggest that at least one auction be a forward auction, where instead the strike price is fixed and bidders bid on the premium. The advantages and disadvantages of these two formats are discussed in the companion document on relevant auction theory. Because the first auction is likely to be a reverse auction, in this document we describe the auction design in terms of the reverse auction and we discuss the forward auction in Appendix 1.

3. Clock Auctions

A clock auction is a dynamic auction that consists of multiple rounds. In each round, the auctioneer announces a new strike price and every bidder state the quantity he/she is willing to buy at that price. The strike price then decreases until aggregate demand is less than or equal to supply. The first price at which aggregate demand is less than or equal to supply is called the clearing price, and every bidder who is still bidding wins the amount he/she is bidding on at that price.

This type of dynamic process encourages price discovery. After each round, bidders see information about other bidders' demand and can use this information to adjust their estimates of the value of each put option.

3.1. Deposits

In order to ensure that bidders are credible and serious, a bid deposit should be required by each bidder. A bidder's bid deposit will determine the maximum quantity for which he/she will be allowed to bid during the auction. The required deposit should be the product of a specified price (set by the PAF) and the maximum quantity for which the bidder wants to bid. For instance, if a bidder wants to bid on a quantity of 100,000 put options and the PAF has set a fixed price of $.05 per unit for the deposit, the required deposit would be $5,000. The deposit should be large enough that the bidder is unlikely to default on his/her obligation, but small enough that it does not create unnecessary barriers to entry. A reasonable deposit might be 25% of the premium in the auction. This deposit is credited toward the payment of a winning bidder at the end of the auction, provided that he/she pays the balance. This deposit is refunded promptly to a losing bidder after the auction.

3.2. Reserve Price

The reserve price is announced before the auction and, in a reverse auction would also be the starting price. This price is the highest strike price that the PAF is willing to accept. The fixed premium should also be announced before the auction.

3.3. Price Decrements

Price decrements should be specified by the auctioneer before the start of the round, or possibly a few rounds in advance. Especially in early auctions, when there is a lot of uncertainty about the price, it may be advantageous not to announce all price decrements at the start of the auction, so as to retain the flexibility to adjust the pace of the auction. The auctioneer can announce prices for the first couple of rounds before the start of the auction and then announce price decrements for the remaining rounds based on the current aggregate demand. For instance, as prices decrease, if aggregate demand is decreasing too fast, the auctioneer may want to decrease the bid decrements to enable more price discovery.

The price decrement can either be announced as a percentage of the current price (e.g. if the current price is $10, a 20% bid decrement would decrease the price to $8 in the next round), or

as an absolute amount (e.g., $1 or $0.5). For the purposes of this auction, it would be more convenient to set absolute decrements.

3.4. Bidding Procedures

In a clock auction, the entire supply is auctioned simultaneously. In each round, the auctioneer announces a strike price. Bidders respond by bidding a quantity of options that they would be willing to buy at that strike price (for the fixed premium). Bidders cannot bid for quantities that exceed the amount specified by their initial deposits or other limits imposed by the activity rule (see section 3.6).

In a clock auction the strike price ticks down in successive rounds, in response to excess demand. This procedure allows the auctioneer to explore the aggregate demand curve without asking bidders for unnecessary information.

3.4.1. Simple Clock Bidding

In a clock auction, bidders can submit bids in response to current round prices. For instance, if the round's strike price is $10, the bidder can submit a bid for 10,000 put options at that price. Alternatively, a bidder can place a bid for zero put options at the round's price to signal their desire to exit the auction.

The clock format does not guarantee that all available put options will be sold when the auction concludes. In particular, some put options may remain unsold; this is referred to as *undersell*. When bidders only submit simple clock bids and the auctioneer uses large decrements, the risk of undersell may be relatively high. If the round's price decrement proves to be too large, then many bidders could drop out of the auction at once, leading to undersell at the final clock price.

An alternative to simple clock bids is to allow intra-round bids; see Appendix 3 for details.

3.4.2. Proxy Bidding

It is best practice to allow bidders who don't care to participate in the live auction to submit proxy bids. These are bids at prices one or more rounds beyond the current round price, signifying that the bidder is willing to buy some quantity as long as the price remains within the amount of the proxy bid. They effectively enable bidders to participate in this auction as if it was a sealed-bid auction. As long as the round price has not reached the proxy bid price, bidders should be able to adjust these bids. For example, if the reserve price is $10, a bidder may submit the following proxy bids at the beginning of the auction:

Price	Bidder's Bid
$10	100,000
$9	90,000
$8	0

These bids indicate that: (1) as long as the strike price is strictly greater than $9, the bidder wants 100,000 put options; (2) if the strike price is less than or equal to $9 and strictly greater than $8, the bidder wants 90,000 put options; and (3) if the strike price is less than or equal to $8, the bidder does not want any put options. During clock rounds, these bids will automatically be submitted for him. As long as the round price is strictly greater than $9, the bidder can change his/her bid corresponding to $9; for instance, he/she may increase it to 95,000.

3.5. Supply

For a given auction, the PAF can either fix a quantity of put options or fix a budget. If the PAF fixes a quantity of put options, the supply is equal to that quantity and does not depend on the strike price.

If the PAF fixes a budget for the auction (which is expected), the supply is a function of the strike price. As the strike price goes down, more put options are added to the auction; in other words, the budget is "recycled" into the auction as each put option becomes cheaper for the facility. In particular, if the strike price is s and the premium is p, the cost of each put option to the facility is s-p. Given a budget of B, the facility can provide at most $B/(s-p)$ put options. Thus, as the strike price s decreases, the supply increases. On the other hand, as the strike price decreases, the aggregate demand will (weakly) decrease, because of the activity rule described in Section 3.6; the auction clears when the strike price is sufficiently low that the aggregate demand is less than or equal to the supply at that strike price. The use of a constant-budget supply curve is illustrated in the example of Section 3.10, below.

3.6. Activity Rule

An activity rule is necessary to make a dynamic auction informative and to avoid "bid sniping". Best practice in high-stakes auctions is to include an appropriate activity rule. We recommend an activity rule requiring that bidders can only maintain their demand or decrease their demand for put options as the strike price descends. (Such a restriction should also be consistent with bidders' true demands.) Bidders who did not bid for a large quantity in the early rounds will not be able to bid for a large quantity later on, inducing them to bid in a more informative way.

3.7. Closing Rule and Winner Determination

The auction ends when aggregate demand is less than or equal to supply. When this happens, the PAF is offering enough put options to satisfy the bids of all active bidders. The winning bidders are the bidders who still have active bids when supply exceeds demand, and each of these bidders is awarded his or her current quantity. The remaining put options are not allocated in this auction and the remaining budget is used for a subsequent auction. Every winning bidders pays the fixed premium to receive a put option with the same strike price; this is the highest strike price at which aggregate demand is less than or equal to supply.

3.8. Defaults

If a bidder defaults on his/her obligation, the bidder should be required to pay a penalty that is taken from his/her bid deposit. The deposit requirement should be set at least as large as the penalty that a defaulting bidder would be required to pay. Following a default, the put options should remain unsold and the unused budget together with the penalty can be used in future auctions.

3.9. Information Policy

An important consideration in a dynamic auction format is how much information about opponents' bids to reveal to bidders between rounds. We recommend revealing aggregate demand at the end of each round, but not any bidders' individual bids. In particular, the auctioneer reveals the total demand for put options at the round price. This reveals sufficient information for bidders to update their bidding, based on others' valuations in the auction, but not so much information as to facilitate collusion.

3.10. Example

Consider the following example. The PAF has a budget of $800,000 to sell in the reverse auction. The premium is fixed at $0.50. The reserve price is $10. The auction proceeds as follows:

Round	Price	Bid of Bidder 1	Bid of Bidder 2	Bid of Bidder 3	Supply
1	$10	100,000	100,000	100,000	84,210
2	$9	80,000	70,000	90,000	94,117
3	$8	40,000	0	55,000	106,666

The final price in this auction is $8, since this is the first price where supply is less than or equal to demand. Bidder 1 wins 40,000 put option and bidder 3 wins 55,000 put options. The winning bidder then pays the premium to the PAF, so Bidder 1 pays $6,000 and bidder 3 pays $8,250. There is undersell of 11,666 put options which corresponds to $87,495; in other words, $87,495 is the amount of the budget that was not allocated as a result of the undersell.

3.11. Bidding Caps

The PAF may want to encourage competition in a given market. In this case, the PAF may want to implement a cap that would prohibit a single bidder from winning more than some percentage (e.g., 25%) of the total initial supply of put options. With a bidding cap, all bidders are limited in the quantities for which they can bid. It is important to make sure that the cap is

not too small, because a small cap may discourage some projects or bidders from participating in the auction.[2]

3.12. Set Aside or "Carve Out"

To provide preferential treatment to some projects (e.g., projects in low-income countries), the PAF may decide to set aside or "carve out" a portion of the supply and offer it as a separate product in the same auction. In particular, there would be two products (i.e., types of put options) in the auction: the "carve out" product that only applies to certain projects (e.g., projects in low-income countries) in the sense that such a put option can only be used for methane abatement that came from those projects, and the "general" product that applies to all approved projects. Bidders would be able to bid on both the "carve out" supply and the general supply, ensuring that the strike price of the "carve out" is never lower than the price of the general product, because the "carve out" product comes with more restrictions Each product would have its own strike price, which would decrease in any round where there was excess demand for that product. Appendix 2 describes how the clock auction format works in a setting with a "carve out" and with a general product.

[2] For any tentatively-selected cap, it would be worthwhile to examine the plausible projects to be bid for in the auction, and to make sure that plausible projects are not cut out by a given cap.

4. Other Implementation Issues

4.1. Online Platform

The auction should be run on an online platform accessible through standard web browsers (Internet Explorer, Safari, etc.) on the internet. The platform should utilize standard security measures, e.g. a digital certificate to encrypt communications and authenticate the web site) that are typically used for online banking. This platform would ensure that bids are submitted in a way that is secure, and would be easy for bidders to use.

Each round would last for a set duration, e.g., for 15 minutes. During this time, a bidder can submit bids or modify a bid that he/she already submitted in this round. Once this time is up, bidders are no longer able to submit or modify bids. There is a brief recess between rounds, to enable bids to be processed and for bidders to plan their next bids or get approval to place future bids. Until the round results have been posted, bidders can be given the opportunity to call in and place bids, in case a technical issue prevented them from bidding during the round.

4.1.1. Auction Manager

This platform would provide the PAF with a variety of tools to conduct the auction. The auction manager would be able to set the schedule of the auction, set up and manage bidders' accounts, announce new rounds, update bid decrements and monitor the auction. In rare cases, the auction manager would be able to place bids on behalf of the bidder, and if this happens, it should be made very clear in the system's audit log.

4.1.2. Trustee

The software may allow independent observers to observe the auction to ensure that the auction is conducted fairly and within the rules. An observer would have read-only access to the entire auction system and could help reassure bidders that the PAF is not manipulating the auction in any way. Given the situation, it may well be adequate for a World Bank employee to have this level of access and to certify the conduct of the auction.

4.1.3. Bidders

The auction platform provides a convenient and secure way for bidders to submit bids. In each round, bidders are able to securely place and edit bids through a straightforward interface. The software should communicate clearly how to submit bids and should enforce the requirements on bids (such as the activity rule) in real time. Bidders should also be able to review the results of the previous rounds (aggregate demand and prices). At the end of the auction, the system should determine who won what and report the results to the bidders.

The system could provide one or multiple logins to bidders from the same firm. This would allow multiple representatives to participate in the auction and provide redundancy in case a representative is stuck in an elevator, etc.

4.2. Collusion

Collusion can be an issue in any auction. It should be stated clearly in the auction's Terms and Conditions that collusion is a violation of the auction rules and will have consequences. Bidders who engage in collusion will forfeit their deposits and be banned from participation in future auctions. One reason to report only aggregate demand to bidders is to make collusive agreements harder to enforce. By conducting the auction online, bidders' identities can be kept anonymous. If there are serious concerns about collusion, in-person conferences should be avoided, so that participants in each auction round are uncertain about who else is participating.

4.3. Training

The PAF should offer training to all auction participants in order to help familiarize participants with the put options and the auction format.

4.3.1. Bidder Conference

The bidder conference is a chance for the PAF to present the rules and the details of the product to the auction participants. This should be held when the auction rules and the details of the put option are finalized, and should be held as soon after the announcement as possible. This conference is open to any interested participants, and the PAF may want to record or telecast the presentations to ensure that participants who cannot get to the conference location can still have access to this important information. Since this conference is held as soon as the rules are finalized, the completed software is not ready to be demonstrated yet, but mock-ups of the various screens could be shown to bidders to help assuage concerns about the complexity of bidding in the auction.

However, as noted above, if there are concerns that there may be collusion among bidders, then a bidder conference (or any other outreach vehicle that enables bidders to learn each other's identities) should be avoided. Instead, outreach and training should occur via individualized methods, such as screencasts, described next.

4.3.2. Screencast

Once the auction software is completed, the PAF should hold a screencast online to demonstrate the software and its features. This will only be open to bidders who have requested to participate in the auction. This screencast gives the PAF the opportunity to introduce various bidders to the different screens of the auction, and to demonstrate how an auction would proceed.

4.3.3. Mock Auctions

The final phase of training is the mock auction. Usually held close to the auction itself, bidders are given the opportunity to bid in a mock auction, against other bidders and representatives of the PAF. The set-up of the mock auction should be as similar to the set-up of the actual auction as possible.

4.4. Post-Auction Information

At the conclusion of the auction, we recommend that the PAF release the identities of the winning bidders (so as to facilitate resale) and the final price. Optionally, the PAF might disclose the quantities won by each of the bidders. We do not recommend that information about specific bids be released.

4.5. Project Schedule

Once the auction is announced, the first step is to finalize the rules, the details of the put options and any documents bidders will have to fill out to participate in the auction. Once this is done, a conference for bidders should be held as soon as possible. This allows firms that are interested in participating to learn enough about the auction to decide whether or not they actually want to participate. At the same time, the customization of auction software and the associated system documentation should commence.

After the bidder conference, there should be a qualification period, a period of time of about a month when firms are able to apply to bid in the auction. This period could be longer if the qualification process bidders have to go through to participate is particularly complicated, which we do not anticipate here. At this time, bidders should tell the PAF how much they expect to deposit, although the deadline for submitting the deposit should not be in this period.

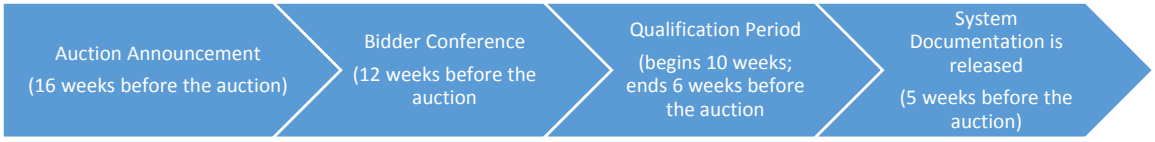
After the qualification period (so perhaps a month after the bidder conference), the applications would be processed and bidders would be notified that their applications to participate have been approved. They would then be notified of arrangements to submit their deposits, and of the deposit deadline. When available, the system documentation should be shared with bidders. A few weeks after this, a screencast should be held to supplement bidders' reading of the manual. During this period, additional training could be offered (for instance, some early mock auctions or face-to-face training) for bidders who are very uncomfortable with the process.

Finally, shortly before the auction, login credentials should be distributed to bidders. Perhaps a week before the auction, bidders should be invited to participate in a mock auction. This gives bidders an opportunity to actually use the auction software before the real auction. The mock auction should be viewed as the most effective training exercise, so bidders' participation should be strongly encouraged.

The auction itself will probably be a fairly short event, at most 10 to 15 rounds. This could certainly be completed within a single day, perhaps even within a morning. Each round should be 10–20 minutes in length, followed by a 10 minute recess. During the recess, bidders learn aggregate demand and the next round's prices are announced. At the end of the auction, once the results are confirmed, they should be announced publicly and payments should be collected within a few weeks.

Given the anticipated announcement in mid-September, we expect that a procedure along these lines could lead to the first live auction being held in January or February. The bidder

conference or other outreach would be held in late September or early October. The qualification period would last until the middle of November, and the screencast would occur either in December or January (avoiding the December holidays). Extra software features such as support for multiple languages would increase the time needed for software readiness. However, preparation of a software platform containing the ordinary level of features for this type of auction is consistent with these time scales. Indeed, usually the limiting factors in getting to the first auction are the times needed for outreach to the potential bidders and the qualification of the bidders.



5. Conclusions

The clock auction format is simple and transparent, and (via an appropriate user interface) can be made appealing to bidders. It is flexible enough to accommodate both a forward and reverse auction. Moreover, the clock format can be used to auction one or more types of put options without creating significant complexity or requiring significant retraining for bidders. The PAF can easily run these auctions online, in a way that is secure and makes participation easy and straightforward. It is for these reasons that we recommend the clock auction format described in this document for the PAF.

Appendix 1: Forward Auction

The clock auction format can be easily modified to be a forward instead of reverse auction. The PAF would set the strike price in advance, bidders would bid on the premium instead of the strike price, and the prices would go up instead of down between rounds. The reserve price is now the lowest premium the PAF is willing to accept.

In each round the auctioneer announces a premium to bidders. Bidders then bid the quantity of put options they would be willing to buy for that premium. Bidders can use intra-round bidding and proxy bids. A proxy bid indicates the highest premium for which the bidder is willing to buy a quantity of put options. The activity rule does not allow bidders to increase their demand for put options as the premium increases.

The auction ends at the lowest premium for which aggregate demand does not exceed the supply.

Appendix 2: Auction with "Carve Out" Product

The clock format can be used to auction multiple products (i.e., types of put options) simultaneously in the same auction. In the context of the PAF, this is relevant if the PAF decides to set aside or "carve out" part of the supply in an auction for certain projects (e.g., projects in low-income countries). In this case, as described in Section 3.12, there will be two products in the auction: the "carve out" product that is limited to certain projects, and the "general" product that applies to all approved projects. In other words, the "carve out" product is a more restricted put option that can only be used for methane abatement that came from certain projects, whereas the general product can be used for methane abatement from any approved project. In this Appendix, we describe how the reverse clock auction works in this case; we denote the general product and the "carve out" product by G and C respectively. Both products will have the same fixed premium. However, each product will have its own strike price (the price that is bid on during the auction).

A2.1 Deposits

A bidder's bid deposit will determine the maximum quantity for which he/she will be allowed to bid during the auction across both products. The required deposit should be the product of a specified price (set by the PAF) and the maximum quantity for which the bidder wants to bid across both products. For instance, if a bidder wants to bid on a quantity of 60,000 put options of the general product and a quantity of 40,000 put options of the "carve out" product and the PAF has set a fixed price of $.05 per unit for the deposit, the required deposit would be $5,000. Note that by paying a deposit of $5,000, the bidder can allocate a quantity of 100,000 any way he/she wants across the two products (subject to the activity rule); for instance, he/she can bid for 80,000 put options of the general product and 20,000 put options of the "carve out" product, or he/she can bid for 0 put options of the general product and 100,000 put options of the "carve out" product.

A2.2 Reserve Prices

Each product should be assigned its own reserve price, the highest strike price the PAF is willing to accept. The procedure will be clearest if both products are assigned the same reserve price, i.e., the same starting price of the auction.

A2.3 Bidding Procedure

Both products are auctioned simultaneously. Each product has a different strike price which decreases independently as the auction proceeds. The auctioneer decreases the price of a product if the aggregate demand for that product exceeded supply in the previous round. Bidders cannot bid for a total quantity of put options across both products that exceed the amount specified by their deposit or any constraints from the activity rule.

A2.2.1 Simple Clock Bids

At the start of each round, the auctioneer announces a strike price for each of the two products. Given these prices, each bidder reports the quantity (i.e., number of put options) that he/she is willing to buy for each product. As in the auction for a single type of put option, intra-round bidding can be used to mitigate the risk of undersell if decrements are too large. This is discussed in Appendix 3.

A2.2.2 Proxy Bidding

Bidders should also be able to submit proxy bids for each of the products. For instance, at the beginning of the auction a bidder could submit bids for 20,000 of the "carve out" product until its price is below $4 and 10,000 for the general product until its price is below $3.50.

A2.4 Supply

The PAF can either fix a quantity of put options or fix a budget for each product. If the PAF fixes a quantity of put options for each product, the supply of the product is equal to that quantity and does not depend on the strike price.

If the PAF fixes a budget for each product, the total budget for the auction is equal to the sum of the budgets of the two products and the supply of each product is a function of its strike price.[3] In this case, the PAF can announce to bidders prior to the auction that the proportion of the budget allocated to each product could be adjusted in mid-auction if participation for the "carve out" product is very low.

A2.5 Activity Rule

Bidders should be restricted from increasing their demand as prices go down. So the total quantity of put options bid on cannot increase as the auction progresses.

Specifically, suppose that G denotes the general product and C denotes the "carve out" product. Consider a bidder who can bid on both products. Let q_G^t be the amount of product G he bid on in round t, and q_C^t be the amount of product C he bid on in round t. Then in the next round, $t+1$, it must be that:

$$q_G^{t+1} + q_C^{t+1} \leq q_G^t + q_C^t \,.$$

So the total quantity of put options any bidder bids on cannot increase as the auction progresses.[4]

[3] Alternatively, the PAF could announce a supply function for each product that depends on both strike prices; however, this approach would complicate the auction.

[4] An alternative is to use a different weight for each product; the weights would represent the relative values of the two products. The more general activity rule would then be that the weighted sum of the quantities of put options is not permitted to increase as the auction progresses.

A2.6 Closing Rule and Winner Determination

The auction for both products ends when, for each product, supply is greater than or equal to aggregate demand. Until then, bidders can bid on any type of put option, even if supply exceeded demand for that option in the previous round. This provides the maximum opportunity for efficient substitution between the two products.

The bidders who are still active when demand drops below supply are the winning bidders and win the quantities they are bidding for. The strike price is the lowest price point between the previous round price and the current round price where demand was less than or equal to supply. So, if during intra-round bidding between round 2 and round 3, for example, demand in all products dropped below supply at the 50% price point, rose above supply for one product at the 60% price point, and dropped back below supply at the round price, then the final strike price would be the 50% price point. Winning bidders are awarded what they bid for at that price point.

A2.7 Example

Consider the following example, where G denotes the general product and C the "carve out". The premium for both products is $0.50. There is a budget of $1,500,000 that is split evenly between the two products; that is, the budget for each product is $750,000. Bidder 3 is only interested in the general product, while the other bidders are interested in bidding on both products.

Rnd	Price of G	Price of C	Bidder 1 for G	Bidder 1 for C	Bidder 2 for G	Bidder 2 for C	Bidder 3 for G	Supply of G	Supply of C
1	$10	$10	100,000	100,000	30,000	30,000	90,000	78,947	78,947
2	$9	$9	90,000	50,000	30,000	30,000	80,000	88,235	88,235
3	$8	$9	60,000	60,000	0	60,000	70,000	100,000	88,235
4	$7	$8	50,000	70,000	0	10,000	50,000	115,385	100,000

Observe that at the price of round 2: for the general product, aggregate demand is greater than the supply; whereas for the "carve out" product, aggregate demand is smaller than the supply. As a result, the price of the general product is decremented, whereas the price of the "carve out" product is not decremented after round 2.

The auction clears at the prices of round 4; at those prices, supply is greater than the aggregate demand for each of the products. Bidder 1 is awarded 50,000 put options of the general product (with strike price $7) and 70,000 of the "carve out" product (with strike price $8). Bidder 2 wins 10,000 put options of the "carve out" product, and Bidder 3 wins 50,000 put options of the general product.

Appendix 3: Intra-Round Bidding

A3.1. Intra-Round Bidding

An alternative to the simple clock bidding described in this document (e.g., in Section 3.4.1) is to allow intra round bidding. This and all other references to "intra-round bids" are not formally part of our design recommendations, as intra-round bidding is a proprietary technique which would only be available if implemented on Power Auctions' software platform. That said, if consistent with the implementation decision, intra-round bidding enables the auctioneer to set large bid decrements and run a quicker auction, without creating undersell or causing any sacrifice to efficiency. Intra-round bids have been routinely used in clock auctions in the energy sector for a decade, and they are part of the announced design for the Federal Communications Commission's upcoming incentive auction.

Intra-round bids are bids between the previous round's price and the current round's price that indicate at exactly what prices the bidder wants to reduce his/her demand. They allow bidders to better express their preferences and they help to reduce the incidence of undersell. They function similarly to sealed bids in a sealed-bid auction, except that with intra-round bids, bidders are only being asked to submit sealed bids for prices in the interval between the previous round's price and the current round's price. If many bidders submit intra-round bids, their use can reduce the quantity of options that end up unsold. However, even with intra-round bids, there is still the possibility of undersell. The use of intra-round bids is illustrated below with an extension of the example from Section 3.10.

A3.2. Example

Recall the example from Section 3.10:

Round	Price	Bid of Bidder 1	Bid of Bidder 2	Bid of Bidder 3	Supply
1	$10	100,000	100,000	100,000	84,210
2	$9	80,000	70,000	90,000	94,117
3	$8	40,000	0	55,000	106,666

With only standard bids, the auction ends at a strike price of $8 and there is undersell of 11,666 put options.

We now illustrate how intra-round bidding may decrease the risk of undersell in this example. In particular, suppose the bidders submit the following intra-round bids between round 2 and 3:

Bidder 1

$8.75	40,000

Bidder 2

$8.90	60,000
$8.60	30,000
$8.45	5,000
$8.05	0

Bidder 3

$8.50	55,000

These bids function like sealed bids, but in a much smaller interval. The clearing price is the highest strike price at which aggregate demand is less than or equal to supply. In this example, when the strike price is $8.45, aggregate demand drops to 100,000, whereas the supply is 100,628. So the final strike price is $8.10. Bidder 1 receives 40,000 put options, bidder 2 receives 5,000 put options and bidder 3 receives 55,000 put options; therefore, there is undersell of 628 put options or $4,922.60, which is significantly lower than the undersell in the case without intra-round bidding.

A3.3. Intra-Round Bidding in an Auction with a "Carve Out" Product

Intra-round bidding is more complicated in the multiple product case. Since there are multiple products with different prices, it is not always clear which intra-round bid should be considered first. To illustrate this, suppose that between round 1 and round 2, the strike price for one product decreases from $9 to $8 and the strike price for the other product decreases from $8 to $7.50. If a bidder submits an intra-round bid decreasing his/her demand for the first put option at $8.50 and an intra-round bid decreasing his/her demand for the second put option at $7.80, it is unclear which of those bids occurred first. The auction may end or not depending on when these bids are processed.

In order to prioritize intra-round bids for different products, it is best practice to use "price-points." The price chosen by each bidder in an intra-round bid is expressed as a percentage of the price decrement. So in the above example, the intra-round bid for the first put option occurred at the 50% price-point, while the intra-round bid for the second put option occurred at the 40% price point. Two bids that occurred at the same price point are treated as occurring simultaneously, while if two bids are submitted at different price points for each product, the

bid that was submitted at the lower price point is treated as the earlier of the two bids. We illustrate these bids using the example from section A2.7.

A3.4. Example with a "Carve Out" Product

Recall the example from Section A2.7. There are two products, each with a budget of $750,000:

Rnd	Price of G	Price of C	Bidder 1 for G	Bidder 1 for C	Bidder 2 for G	Bidder 2 for C	Bidder 3 for G	Supply of G	Supply of C
1	$10	$10	100,000	100,000	30,000	30,000	90,000	78,947	78,947
2	$9	$9	90,000	50,000	30,000	30,000	80,000	88,235	88,235
3	$8	$9	60,000	60,000	0	60,000	70,000	100,000	88,235
4	$7	$8	50,000	70,000	0	10,000	50,000	115,385	100,000

The auction ends when the price of G is $7 and the price of C is $8. Now suppose that bidders have submitted the following intra-round bids between rounds 3 and 4:

Product G

Bidder	Price	Quantity
1	$7.50	50,000
3	$7.70	50,000

Product C

Bidder	Price	Quantity
1	$8.50	70,000
2	$8.80	10,000

The 50% price point is the first price point at which aggregate demand does not exceed supply for each of the products simultaneously, so the final strike prices are: $7.50 for the general product and $8.50 for the "carve out" product. Bidders are given the quantities that they bid for at those prices.

A3.5. Activity Rule in an Auction with a "Carve out" Product

Intra-round bids are required to satisfy the activity rule at every price point, not just at the round prices. So a bidder would not be able to submit a bid that decreased his/her demand at the 70% price point for one type of put option, and then increased his/her demand for the other at the 80% price point, since at the 70% price point the bidder's aggregate demand decreased, and at the 80% price point the bidder tried to increase his/her demand, which would violate the constraint.